STRENGTH OF MATERIALS II

Eccentric Normal Force

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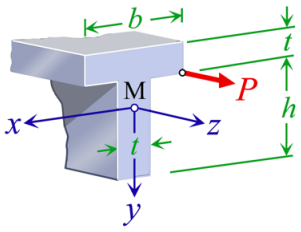
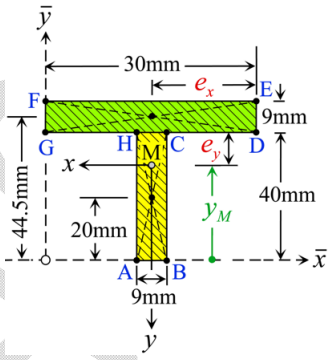
Question : A concentrated single load $P=8\,\text{kN}$ is applied on the cross-section of a profile as given in Figure. The allowable stress on the section is $\sigma_{\text{all}}=120\,\text{MPa}$. Check the security of the section. $(b=30\,\text{mm}, h=40\,\text{mm}, t=9\,\text{mm})$.



Solution:

The area A , center of gravity y_M and moment of inertias (I_x, I_y) of the cross-section are obtained using the geometric parameters,



$$A=30\times9+40\times9=630\,\text{mm}^2$$

$$y_M=\frac{\sum_{i=1}^{2}y_iA_i}{A}=\frac{(30\times9)44.5+(40\times9)20}{630}=30.5$$

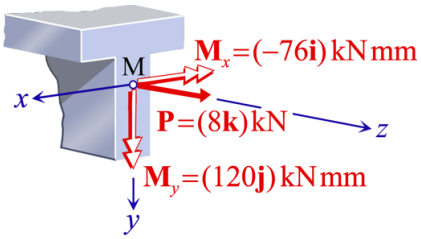
$$I_x=\left[\tfrac{1}{12}\left(9\times40^3\right)+9\times40\left(30.5-20\right)^2\right]$$

$$+\left[\tfrac{1}{12}\left(30\times9^3\right)+30\times9\left(30.5-44.5\right)^2\right]=142433\,\text{mm}^4$$

$$I_y=\tfrac{1}{12}\left(40\times9^3\right)+\tfrac{1}{12}\left(9\times30^3\right)=22680\,\text{mm}^4$$

The eccentricity of the tension force is $e_x=15\,\text{mm}$ and $e_y=9.5\,\text{mm}$.

The applied force is reduced to the axis of the beam as,



$$N=8\times10^3\,\text{N}$$

$$M_x=-Pe_y=-76\times10^3\,\text{Nmm}$$

$$M_y=-Pe_x=120\times10^3\,\text{Nmm}$$

The normal stress distribution is determined using the reduced force components,

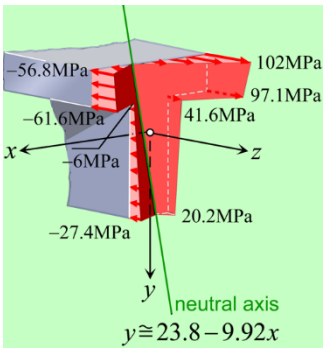
$$\sigma_z=\frac{N}{A}+\frac{M_x}{I_x}y-\frac{M_y}{I_y}x$$

$$=\frac{8\times10^3}{630}+\frac{\left(-76\times10^3\right)}{142433}y-\frac{120\times10^3}{22680}x$$

$$\cong12.7-0.533y-5.29x$$

The neutral axis is the line where the normal stresses are equal to zero,

$\sigma_z = 0 \quad \Rightarrow \quad y \cong 23.8 - 9.92x$



The normal stresses on the cross-section is obtained using the formulae of normal stress distribution,

$\sigma_A = \sigma_z\big|_{\substack{x=4.5 \\ y=30.5}} \cong -27.4 \text{ MPa} \quad ; \quad \sigma_C = \sigma_z\big|_{\substack{x=-4.5 \\ y=-9.5}} \cong 41.6 \text{ MPa}$

$\sigma_E = \sigma_z\big|_{\substack{x=-15 \\ y=-18.5}} \cong 102 \text{ MPa} \quad ; \quad \sigma_G = \sigma_z\big|_{\substack{x=15 \\ y=-9.5}} \cong -61.6 \text{ MPa}$

$\sigma_B = \sigma_z\big|_{\substack{x=-4.5 \\ y=30.5}} \cong 20.2 \text{ MPa} \quad ; \quad \sigma_D = \sigma_z\big|_{\substack{x=-15 \\ y=-9.5}} \cong 91.7 \text{ MPa}$

$\sigma_F = \sigma_z\big|_{\substack{x=15 \\ y=-18.5}} \cong -56.8 \text{ MPa} \quad ; \quad \sigma_G = \sigma_z\big|_{\substack{x=4.5 \\ y=-9.5}} \cong -6 \text{ MPa}$