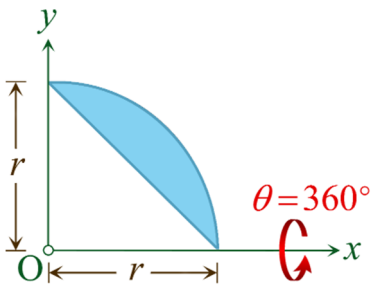
STATICS

Center of Gravity

Dr. Umit N. ARIBAS

Question : Using the theorem of Pappus-Guldinus, determine location of the centroid of the shaded area.

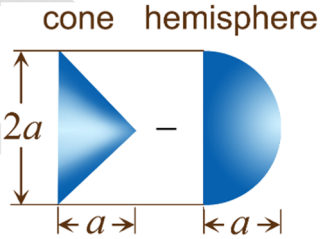


Solution :

Separate the area into simple geometric shapes, which will make the mathematical operations easier.

The analytical expressions of volume:

$V_{\text{cone}} = \frac{1}{3}\pi r^3 \quad ; \quad V_{\text{hemisphere}} = \frac{2}{3}\pi r^3$



The volume of the body is obtained using the analytical expressions:

$V_x = \frac{2}{3}\pi r^3 - \frac{1}{3}\pi r^3 = \frac{1}{3}\pi r^3$

The volume of the body based on theorem of Pappus-Guldinus :

$V_x = 2\pi \bar{y} A = (2\pi)\bar{y}\left[\frac{1}{2}r^2\left(\frac{1}{2}\pi - 1\right)\right] = \frac{1}{2}\pi r^2(\pi - 2)\bar{y}$

***Note: 2π is used since the rotation for 360° corresponds to it. $\bar{y}$ is the center of the gravity of given area which can be calculated by subtracting a triangle from a quarter disc. Similarly, the area *A*.**

The location of the centroid of the shaded area:

$$\bar{y} = \frac{2r}{3(\pi - 2)} \cong 0.584r$$